

# *Working Mathematically*

## **SAMPLE ACTIVITIES**

for the Stage 4 Mathematics Syllabus

0	0	0	.	<b>0</b>	<b>0</b>	<b>1</b>
0	0	0	.	<b>0</b>	<b>1</b>	0
0	0	0	.	<b>1</b>	0	0
0	0	<b>1</b>	.	0	0	0
0	<b>1</b>	<b>0</b>	.	0	0	0
<b>1</b>	<b>0</b>	<b>0</b>	.	0	0	0

# NUMBER

**H. McMaster & M. Mitchelmore**

The size of a number



In some places, people need to know changes in temperature and wind speed to avoid being frostbitten.

At this Golf and Country Club, people go cross-country skiing in the winter but sometimes it can get too dangerous to be outside.

The temperature felt by bare skin is determined not only by the temperature of the air but also by the speed of the wind. This temperature is called the wind chill.

The table below shows how wind chill (measured in °C) varies with air temperature and wind speed.

		Air temperature (°C)												
		0	-1	-2	-3	-4	-5	-10	-15	-20	-25	-30	-35	-40
Wind speed (km/h)	6	-2	-3	-4	-5	-7	-8	-14	-19	-25	-31	-37	-42	-48
	8	-3	-4	-5	-6	-7	-9	-14	-20	-26	-32	-38	-44	-50
	10	-3	-5	-6	-7	-8	-9	-15	-21	-27	-33	-39	-45	-51
	15	-4	-6	-7	-8	-9	-11	-17	-23	-29	-35	-41	-48	-54
	20	-5	-7	-8	-9	-10	-12	-18	-24	-30	-37	-43	-49	-56
	25	-6	-7	-8	-10	-11	-12	-19	-25	-32	-38	-44	-51	-57
	30	-6	-8	-9	-10	-12	-13	-20	-26	-33	-39	-46	-52	-59
	35	-7	-8	-10	-11	-12	-14	-20	-27	-33	-40	-47	-53	-60
	40	-7	-9	-10	-11	-13	-14	-21	-27	-34	-41	-48	-54	-61

When a cross-country skier set out from the clubhouse one morning, the air temperature was -35 °C and the wind speed was 10 km/h.

What was the wind chill when he set out?.....

In the afternoon, the air temperature rose by 5 °C and the wind speed rose by 30 km/h. What was the wind chill then? .....

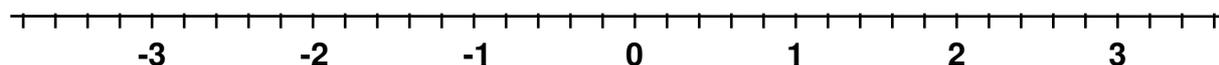
Mark this number line with the morning and afternoon wind chill:

Did the wind chill rise or fall? .....

Use the number line below to find the difference between the following temperatures:

1)  $0.2\text{ °C}$  and  $3.2\text{ °C}$  = .....                      3)  $-0.2\text{ °C}$  and  $-3.2\text{ °C}$  = .....

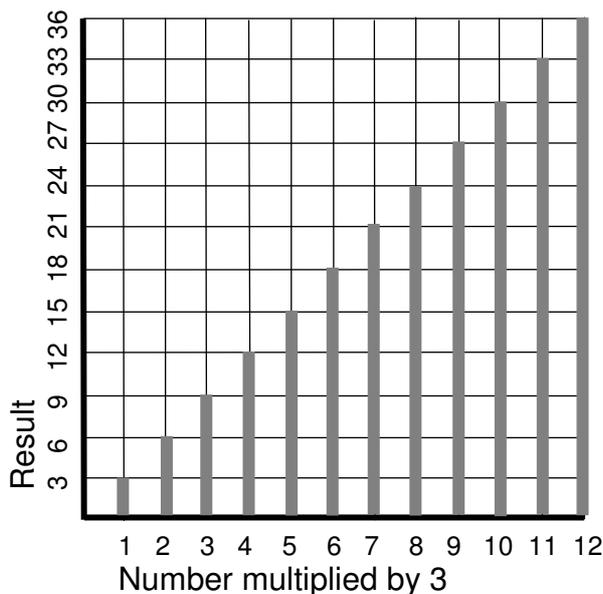
2)  $-0.2\text{ °C}$  and  $3.2\text{ °C}$  = .....                      4)  $0.2\text{ °C}$  and  $-3.2\text{ °C}$  = .....



Patterns in times tables

Below is the 3 times table and a column graph of this times table. The **horizontal axis** (across) shows the number that is multiplied by 3. The **vertical axis** (up and down) shows the answer to the multiplication. Rule a line that goes through the end point of each column in the graph below. The relationship between the number that is multiplied and the result of the multiplication is a **linear relationship** because the results lie in a straight line.

- $3 \times 1 = 3$
- $3 \times 2 = 6$
- $3 \times 3 = 9$
- $3 \times 4 = 12$
- $3 \times 5 = 15$
- $3 \times 6 = 18$
- $3 \times 7 = 21$
- $3 \times 8 = 24$
- $3 \times 9 = 27$
- $3 \times 10 = 30$
- $3 \times 11 = 33$
- $3 \times 12 = 36$

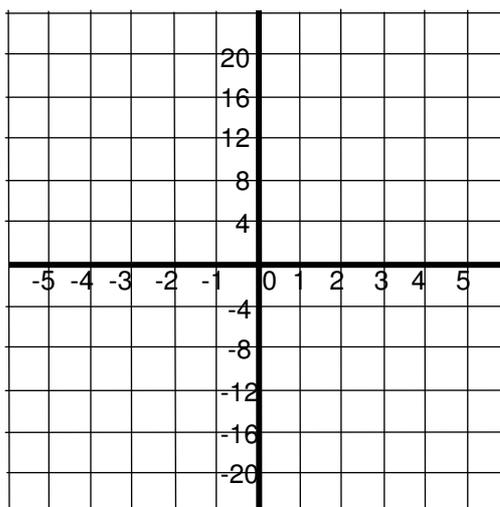


Why do the answers to the multiplications above keep going up by 3?

.....

Write answers to each multiplication below. Then graph your results.

- $4 \times 5 = \dots\dots\dots$
- $4 \times 4 = \dots\dots\dots$
- $4 \times 3 = \dots\dots\dots$
- $4 \times 2 = \dots\dots\dots$
- $4 \times 1 = \dots\dots\dots$
- $4 \times 0 = \dots\dots\dots$
- $4 \times (-1) = \dots\dots\dots$
- $4 \times (-2) = \dots\dots\dots$
- $4 \times (-3) = \dots\dots\dots$
- $4 \times (-4) = \dots\dots\dots$
- $4 \times (-5) = \dots\dots\dots$



The answers to the multiplications above keep going down by 4. Why do you think they do this?

.....

.....

Answer these multiplications:

$$3 \times 5 = \dots \quad 5 \times 3 = \dots \quad 4 \times (-5) = \dots \quad (-5) \times 4 = \dots$$

Does the order of the two numbers being multiplied matter? .....

Use this fact to answer the multiplications below.

Then graph your results.

$$(-5) \times 5 = \dots$$

$$(-5) \times 4 = \dots$$

$$(-5) \times 3 = \dots$$

$$(-5) \times 2 = \dots$$

$$(-5) \times 1 = \dots$$

$$(-5) \times 0 = \dots$$

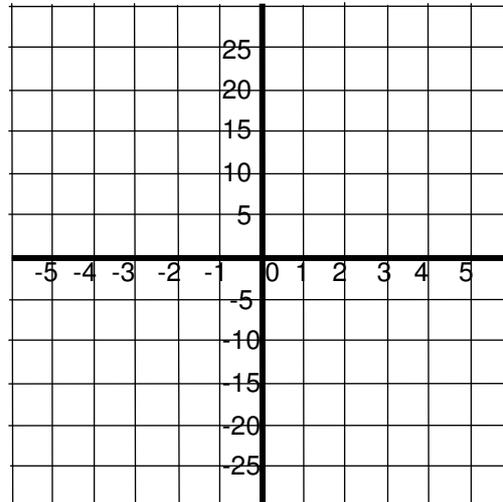
$$(-5) \times (-1) = \dots$$

$$(-5) \times (-2) = \dots$$

$$(-5) \times (-3) = \dots$$

$$(-5) \times (-4) = \dots$$

$$(-5) \times (-5) = \dots$$



In the  $(-5)$  times table above, the numbers keep going..... by.....

The opposite of multiplication is division.

$$(-5) \times 4 = (-20) \quad \text{so} \quad (-20) \div 4 = \dots \quad \text{and} \quad (-20) \div (-5) = \dots$$

$$(-5) \times (-4) = \dots \quad \text{so} \quad 20 \div (-4) = \dots \quad \text{and} \quad 20 \div (-5) = \dots$$

Complete the following sentences by writing positive or negative.

- When a positive number is multiplied or divided by a positive number, the answer is a ..... number.
- When a positive number is multiplied or divided by a negative number, the answer is a ..... number.
- When a negative number is multiplied or divided by a positive number, the answer is a ..... number.
- When a negative number is multiplied or divided by a negative number, the answer is a ..... number.

Answer the following questions.

$$1) (-4) + 4 = \dots \quad 5) (-4) \times 4 = \dots \quad 9) (-4) \div 4 = \dots$$

$$2) (-4) - 4 = \dots \quad 6) 4(-4) = \dots \quad 10) (-4) \div (-4) = \dots$$

$$3) (-4) - (-4) = \dots \quad 7) (-4) \times (-4) = \dots \quad 11) 4 \times (-4) \div (-4) = \dots$$

$$4) (-4) + (-4) = \dots \quad 8) 4 \div (-4) = \dots \quad 12) (-4) \times (-4) \times (-4) = \dots$$

Make these calculations:

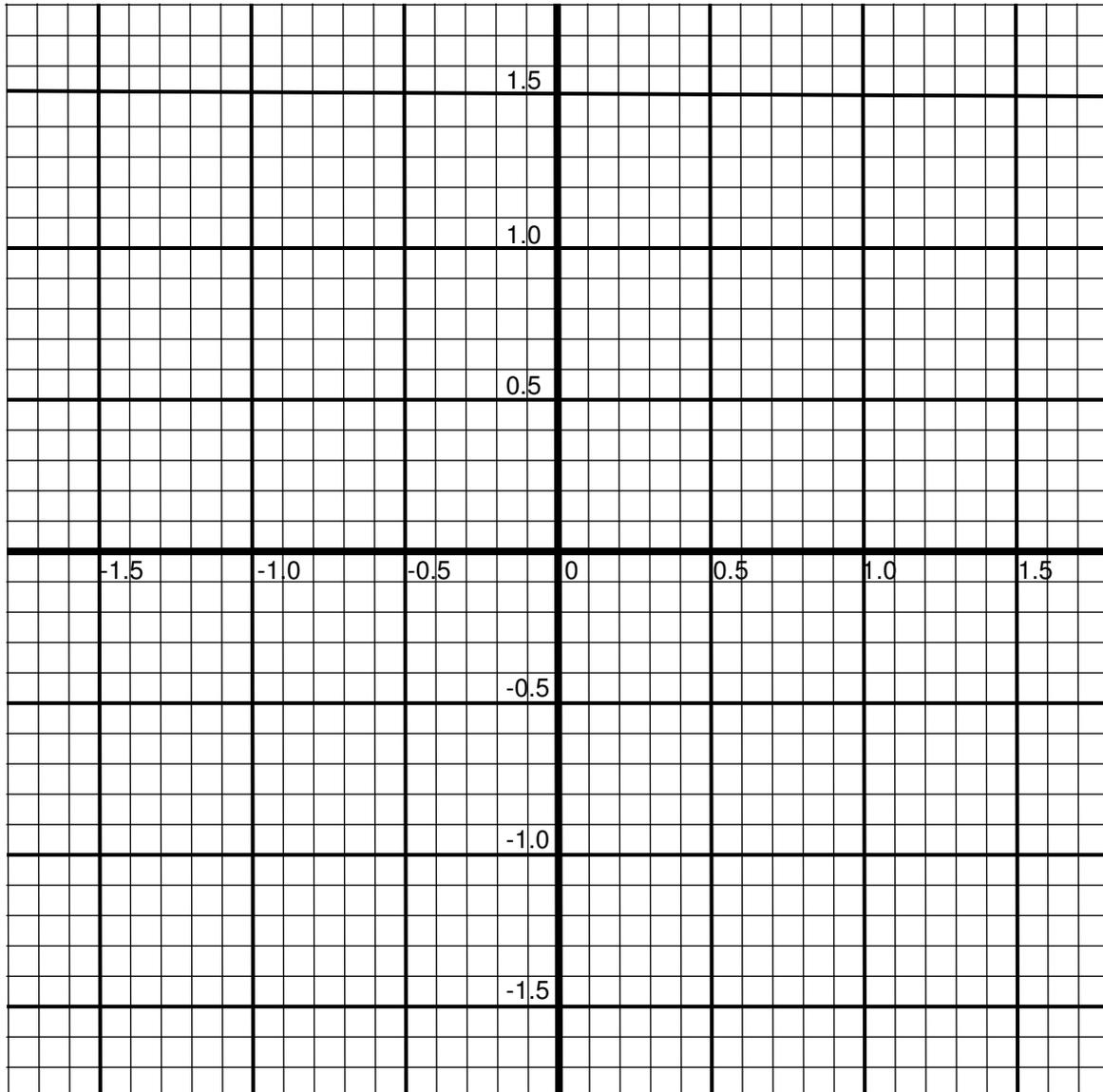
$$(-1.2) \times 0.5 = \dots\dots\dots \quad (-1.2) \times (-0.5) = \dots\dots\dots$$

$$(-1.2) \times 1.0 = \dots\dots\dots \quad (-1.2) \times (-1.0) = \dots\dots\dots$$

$$(-1.2) \times 1.5 = \dots\dots\dots \quad (-1.2) \times (-1.5) = \dots\dots\dots$$

Plot your answers as columns on the graph below.

Then rule a line that goes through the end point of each column.



From your graph, find the approximate values of

$$\begin{array}{ll} (-1.2) \times 1.4 = \dots\dots\dots & (-1.2) \times (-1.15) = \dots\dots\dots \\ (-1.2) \times 0.65 = \dots\dots\dots & (-1.2) \times (-0.9) = \dots\dots\dots \end{array}$$

Check your answers by making these calculations in the space below.

Common multiples

Sand timers



These are sand timers (also called hourglasses). The time they measure is the time it takes for the sand to fall from the top bulb, through the narrow tube, into the bottom bulb.

Once all the sand has fallen to the bottom bulb, the timer can be quickly turned over in order to measure the same time again.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96

One of the timers measures 5 minutes. So by using it over and over again, it measures multiples of 5 minutes.

Lightly shade all the boxes in this number chart that have multiples of 5 written in them.

The other timer measures multiples of 7 minutes.

Use a different type of shading to shade all the boxes in this chart that have multiples of 7 written in them.

Circle all the numbers in the chart that have been shaded twice.

If the time taken to turn over a sand timer doesn't count, what time intervals (up to 96 minutes) can be measured by either the 5 minute timer or the 7 minute timer?.....

How could you get your answer from looking at the number chart?.....

How could you get your answer from the numbers 5 and 7?.....

The time intervals that can be measured by either the 5 minute timer or the 7 minute timer are common multiples of 5 and 7.

A **common multiple** of two (or more) numbers is any number which is a multiple of all the given numbers.

Write down three other common multiples of 5 and 7:.....  
 How many common multiples of 5 and 7 are there?.....

## Fractions and percentages

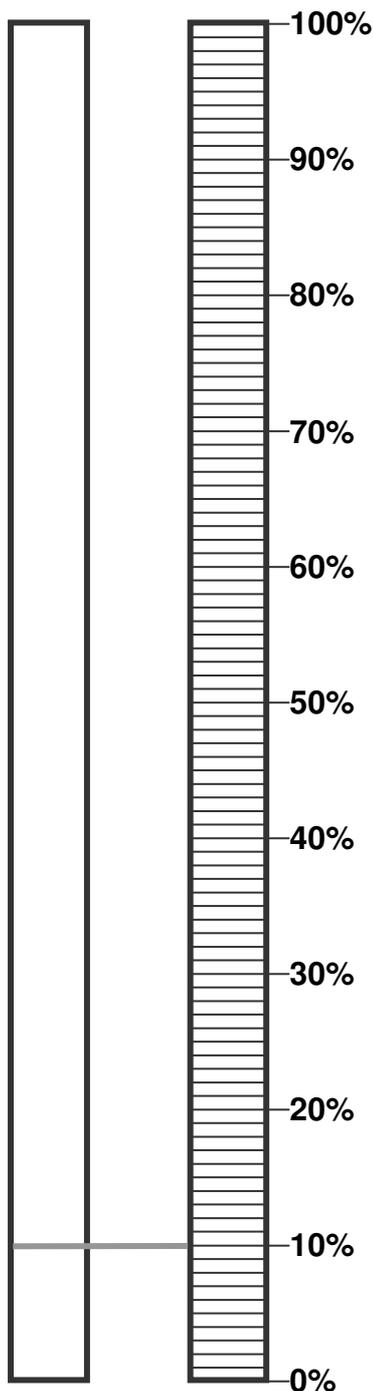
The bar below represents one whole. One whole is 100%.

Using a sharp pencil and ruler, carefully divide the height of the first bar to mark the fractions listed in the table.

By ruling a line straight across to the second bar, you can then find out what each fraction is as a percentage.

Write your answers (to the nearest 1%) in the table.

The first fraction in the table has been done for you.



Fraction	Percentage
$\frac{1}{10}$	10%
$\frac{1}{4}$	
$\frac{1}{2}$	
$\frac{3}{4}$	
$\frac{1}{5}$	
$\frac{2}{5}$	
$\frac{3}{5}$	
$\frac{4}{5}$	
$\frac{1}{3}$	
$\frac{2}{3}$	
$\frac{1}{6}$	
$\frac{5}{6}$	

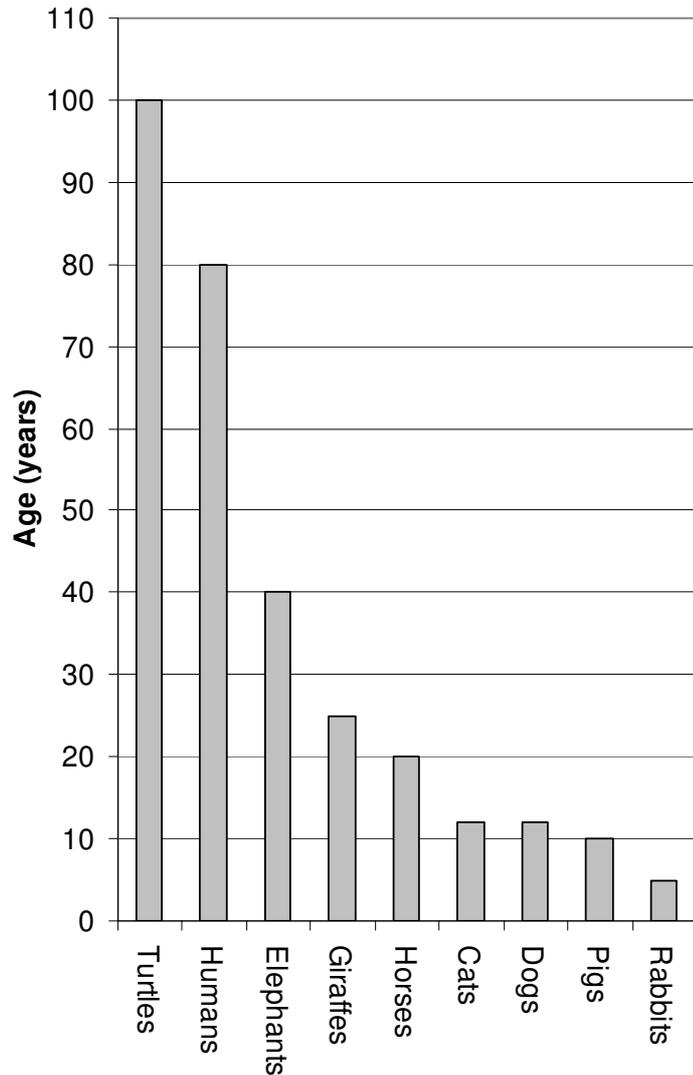
Absolute and relative comparisons

A life span is the length of a life. In wealthy countries, humans have an average life span of about 80 years.

The average life span of humans and various other species are shown in the table and the column graph below.

SPECIES	Average Life Span
Turtles	100
Humans	80
Elephants	40
Giraffes	25
Horses	20
Cats	12
Dogs	12
Pigs	10
Rabbits	5

**Average Life Spans**



- If a person when they reach about 40 is considered to be middle-aged,
- at about what age would a turtle become middle-aged? .....
  - at about what age would a giraffe become middle-aged? .....
  - at about what age would an elephant become middle-aged? .....

On the graph above, mark the beginning of middle-age for each species by drawing a blue line across each column.

How did you decide where to put your marks? .....

.....

If a girl and her horse are both 10 years old, they are the same age in an absolute sense. This is called an **absolute comparison**.

If a girl and her horse have just reached middle-age, they are the same age in a relative sense. This is called a **relative comparison**.

It is a relative comparison because their “age” is related to their life span.

This 50 year old man has a 15 year old horse.

In an absolute sense, which is older (the horse or the man)?.....



On the graph on the previous page, use a pencil to:

- mark the position of a 15 year old horse on the “horse column”
- mark the position of a 50 year old man on the “human column”.

Which is older in a relative sense, the horse or the man?.....

How could you tell this by comparing fractions?.....

.....

How could you tell this from looking at the graph on the previous page?

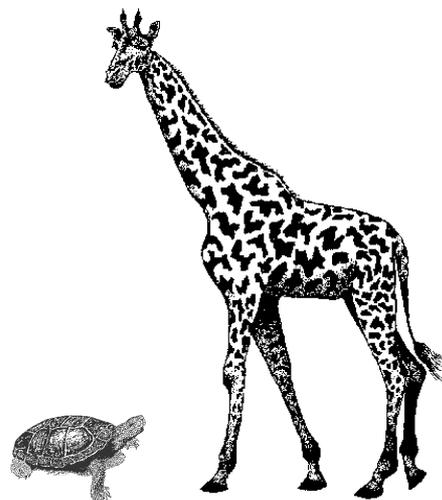
.....

.....

- If a horse is 15 years old,
- what is its relative turtle age?.....
  - what is its relative human age?.....
  - what is its relative elephant age?.....
  - what is its relative dog age?.....
  - what is its relative pig age?.....

What is the relative turtle age of:

- a 10 year old horse?.....
- a 10 year old elephant?.....
- a 10 year old pig?.....
- a 10 year old giraffe?.....
- a 10 year old human?.....



Inverse fractions

In his job, Ken needs to carry a laptop computer with him. His current laptop is too heavy. He wants to buy something lighter.



Ken's current laptop weighs 3.6 kg. He sees an advertisement for a notebook weighing 2.4 kg and a subnotebook weighing only 900 g.

To compare the weight of the notebook with the weight of his laptop, Ken wrote the relationship as a fraction.

Is this a relative or absolute comparison (See Activity 8 -1)?.....

$$\frac{\text{Weight of notebook}}{\text{Weight of laptop}} = \frac{2.4}{3.6} = \frac{24}{36} = \frac{2}{3}$$

So the notebook is 2 thirds of the weight of the laptop.

To calculate the weight of the laptop as a fraction of the weight of the notebook, the weight of the notebook becomes the denominator.

$$\frac{\text{Weight of laptop}}{\text{Weight of notebook}} = \frac{3.6}{2.4} = \frac{36}{24} = \frac{3}{2}$$

So the laptop is 3 halves of the weight of the notebook.  $\frac{3}{2} = 1\frac{1}{2}$

So in other words, the laptop is  $1\frac{1}{2}$  times the weight of the notebook.

The fractions calculated above (ie.  $\frac{2}{3}$  and  $\frac{3}{2}$ ) are inverses of each other.

They are **inverses** because in their simplest form, the numerator of the first fraction is the denominator of the second fraction and vice versa.

The **inverse** of a fraction is also called its **reciprocal**.

What is the product of  $\frac{2}{3}$  and  $\frac{3}{2}$ ?..... (Hint: Multiply them together.)

Do all pairs of inverse fractions have the same product?.....

Calculate the weight of the subnotebook as a fraction of the weight of Ken's laptop. (Note: You must use the same units for both values.)

The subnotebook is.....of the weight of the laptop.

Calculate the weight of Ken's laptop as a fraction of the weight of the subnotebook . Write this fraction in its simplest form.

The laptop is.....times the weight of the subnotebook. (Hint:  $\frac{4}{1} = 4$ )

## Percentage increases and decreases

### Café tips

Darren and Nadina work together in a café. They have agreed to share the tips they receive equally between them.

One day Nadina received \$25 in tips and Darren received \$20 in tips.

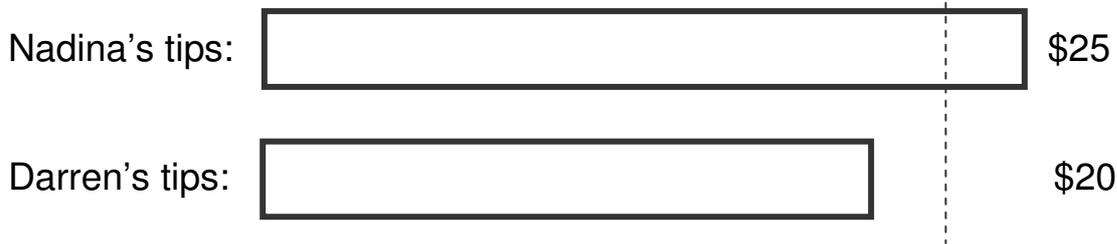
Calculate the amount that Darren received as a percentage of the amount that Nadina received.

So Darren received .....% of what Nadina received.

Calculate the amount that Nadina received as a percentage of the amount that Darren received.

So Nadina received .....% of what Darren received.

The dashed line drawn across the bar diagram shows the amount they should each have when their tips are divided equally between them.



What is the difference between Nadina's tips and Darren's tips? \$.....

To share the tips equally between them, what amount should Nadina give Darren? \$.....

Calculate this amount as a percentage of Nadina's tips.

So Nadina's amount will decrease by .....%

As a result of the sharing, Darren's amount increased by \$.....

Calculate this amount as a percentage of Darren's tips.

So Darren's amount will increase by .....%

Why is the percentage decrease in Nadina's amount smaller than the percentage increase in Darren's amount?.....

.....

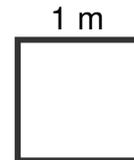
Length and area

Roy made a guinea pig cage by wiring together 4 mesh panels he found at the tip. It has a square base because the 4 side panels are all the same length.

Now Roy's looking for more panels so he can make another guinea pig cage.



The base of the original guinea pig cage is shown in this diagram. The sides are each 1 m long.



What is the perimeter of the base?.....

What is the area of the base? .....

When Roy went to find some more panels, he could only find panels that were 20% longer or 20% shorter than the ones he used before.

What was the length of one of the longer panels?.....

What was the length of one of the shorter panels?.....

If he made a square cage using the longer panels,

- what would be the perimeter of the base?.....
- what would be the area of the base? .....

Compared to the original cage,

- calculate the percentage increase in the perimeter.
  
- calculate the percentage increase in the area of the base.

If he made a square cage using the shorter panels,

- what would be the perimeter of the base?.....
- what would be the area of the base? .....

Compared to the original cage,

- calculate the percentage decrease in the perimeter.
  
- calculate the percentage decrease in the area of the base.

If the original panel had been a different length (eg 2 m instead of 1 m) and the new panels had been 20% shorter or 20% longer than this, would the percentage results be the same as your answers above?.....

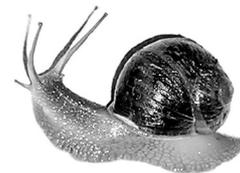
Why or why not?.....  
 .....  
 .....

## Equivalent rates

Gerty the garden snail always travels at a **constant** speed.  
In other words, her speed does not change.

Gerty finished a 10 cm snail race in 20 seconds.  
Calculate Gerty's speed in centimetres per second.

$$\begin{aligned} \text{Speed} &= \dots \div \dots \text{cm/s} \\ &= \dots \text{cm/s} \end{aligned}$$



This means that Gerty travels ..... cm in 1 second.

How many centimetres does Gerty travel in 2 seconds? ..... cm

So Gerty's speed could also be expressed as ..... cm/2s.

How many centimetres does Gerty travel in 6 seconds? ..... cm

So Gerty's speed could also be expressed as ..... cm/6s.

Gerty's speed is constant. So these rates are **equivalent** (ie. the same):

$$0.5 \text{ cm/s} = \dots \text{cm/2s} = \dots \text{cm/6s}$$

How many centimetres does Gerty travel in 60 seconds? ..... cm

So Gerty's speed could also be expressed as ..... cm/60s or ..... cm/min.

How many centimetres does Gerty travel in an hour? ..... cm

So Gerty's speed could also be expressed as ..... cm/h.

Gerty's speed is constant. So these rates are also equivalent:

$$0.5 \text{ cm/s} = \dots \text{cm/min} = \dots \text{cm/h.}$$

As the unit of time (in the denominator) gets larger, does the distance travelled in this unit of time get larger or smaller? .....

How many millimetres does Gerty travel in 1 hour? ..... mm/h

How many metres does Gerty travel in 1 hour? ..... m/h

How many kilometres does Gerty travel in 1 hour? ..... km/h

$$\text{So } 1800 \text{ cm/h} = \dots \text{mm/h} = \dots \text{m/h} = \dots \text{km/h.}$$

As the unit of distance (in the numerator) gets larger, does the number of these units travelled in the time get larger or smaller? .....

Hilda the house mouse runs constantly at 12 km/h.

What is her speed in metres per hour? ..... m/h

Calculate her speed in m/s (to 1 decimal place).

$$\text{Speed} =$$

=

=

=



Inverse rates

Resources required:  
a calculator.

Birth rate

In 2008, the Australian Bureau of Statistics estimated that children were being born in Australia at the rate of 0.0087 births per second.

It doesn't make much sense to talk about a fraction of a birth, so they decided to express the same value in a different way.

0.0087 births/second is  $\frac{87}{10\,000}$  births per 1 second.

$\frac{87}{10\,000}$  births per 1 second = 87 births per 10 000 seconds  
= 1 birth per  $\frac{10\,000}{87}$  seconds.



This is 1 birth every .....seconds (to the nearest second).

Notice that:  $\frac{87}{10\,000}$  is the inverse of  $\frac{10\,000}{87}$ .

So the number of births per second is the inverse of the number of seconds per birth.

Without first converting 0.0087 births/second to a fraction, how could you calculate the number of seconds/birth? .....

Draw the button(s) on your calculator that can you use to find the inverse of a decimal. The inverse is also called the **reciprocal**.

Water use efficiency

Farmer Fred was concerned about the rising cost of the irrigation water he needed to produce his rice crop. So Fred tried to reduce the amount of water he required to grow each tonne of rice.

The table below shows the amount of rice Fred produced in each of the next 5 years and the quantity of irrigation water he used to produce it.

Calculate his water use efficiency (WUE) as the tonnes of rice he produced per megalitre of water he used. Then calculate his WUE as the megalitres of water he used per tonne of rice he produced.

Year	Rice produced (t)	Water used (ML)	WUE (t/ML)	WUE (ML/t)
1	580	928		
2	350	455		
3	380	456		
4	250	275		

Did Fred's use of irrigation water become more efficient? .....

Multiplying and dividing rates

Russell (who lives in Australia) and his brother Randall (who lives in the United States) want to compare the fuel efficiency of their vehicles.

In Australia, fuel efficiency is usually measured as the number of litres of petrol needed to drive 100 km. In the United States it is usually measured as the number of miles you can travel for every gallon of petrol.



Russell and Randall each measured the amount of petrol needed to refill their tank after travelling a measured distance.

Russell drove his Land Rover 500 km before he needed to refill his tank with 70 L of petrol.

Calculate the number of litres Russell used per 100 km.

$$\begin{aligned} \text{Fuel efficiency} &= \dots\dots\dots \\ &= \dots\dots\dots \text{L per 100 km} \end{aligned}$$

Randall drove his Chevrolet 4WD 400 miles before he needed to refill his tank with 25 gallons of petrol.

Calculate the number of miles Randall travelled per gallon of petrol.

$$\begin{aligned} \text{Fuel efficiency} &= \dots\dots\dots \\ &= \dots\dots\dots \text{mpg (miles per gallon)} \end{aligned}$$

To find out whose vehicle was more fuel efficient, they had to use a common unit of measurement.

Russell decided to convert Randall's mpg into L per 100 km.

A mile is 1.61 km. A gallon is 3.8 L.

$$\begin{aligned} &\text{Fuel efficiency of Randall's vehicle} \\ &= \dots\dots\dots \text{mpg} \\ &= \dots\dots\dots \times \dots\dots\dots \text{ km per gallon} \\ &= \dots\dots\dots \times \dots\dots\dots \div \dots\dots\dots \text{ km per L} \\ &= \dots\dots\dots \times \dots\dots\dots \div \dots\dots\dots \div \dots\dots\dots \text{ lots of 100 km per L} \\ &= \dots\dots\dots \text{ lots of 100 km per L (to 4 d.p.)} \\ &= \dots\dots\dots \text{ L per 100 km (to 1 d.p.)} \end{aligned}$$

Whose vehicle was the more fuel efficient? .....

Calculate how many km Randall could drive on one of his tanks of petrol.

$$\begin{aligned} \text{Distance Randall could drive} &= \dots\dots\dots \text{ km} \\ &= \dots\dots\dots \text{ km} \end{aligned}$$

