Working Mathematically in Measurement

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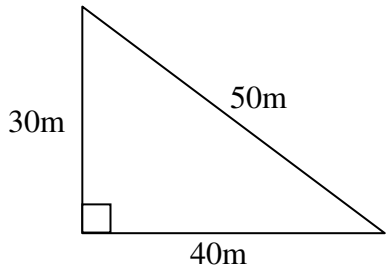
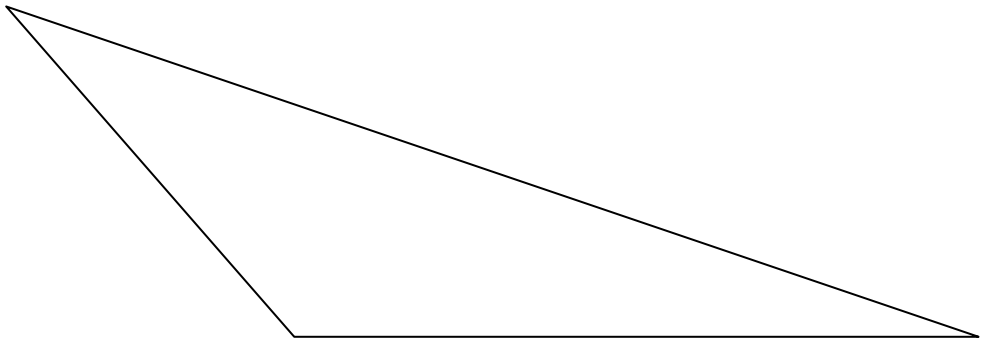
Here are some questions you could use to assess what your students know about the area of a triangle.

1. What is the area of the park in this diagram?



2. What is the area of this triangle?



3. A triangular piece of cake (with uniform thickness) was sliced from one vertex to the midpoint of the opposite side. How do you know that the resulting two pieces of cake are the same size? Use a diagram to illustrate your answer.

4. A triangle has side lengths of 12cm, 10cm and 5cm. Name the side lengths of a triangle that has the same perimeter but a larger area.

If students give 120m as their answer to the first question and have little idea how to answer the other three questions, it could be because they have only learnt formulae. Formulae are difficult for many students to remember and they can easily confuse formulae if they don't know how mathematicians got them in the first place. A student may be able to do standard textbook exercises but not be able to apply their knowledge to a problem in which they don't have the

usual bits of information as a cue for which formula to use. The new syllabus has tried to address these problems by requiring students to "work mathematically".

What is "Working Mathematically"?

"Working Mathematically" appears to be a uniquely Australian expression used to describe how mathematics is used in practice. It includes the processes of questioning, apply strategies, communicating, reasoning and reflecting. The objective is that:

> Students will develop knowledge, skills and understanding through inquiry, application of problem-solving strategies including the selection and use of appropriate technology, communication, reasoning and reflection. (Board of Studies, 2002, p. 12)

The implication is that teachers are expected to follow a process-based approach to mathematics, treating mathematics as a set of interesting challenges or problems rather than as a series of methods and formulae to be learnt for examinations. People who advocate a "working mathematically" approach to teaching believe that students learn mathematics best by making sense of it. The intended consequences are that students find mathematics enjoyable and that they develop self-confidence to use mathematics wherever appropriate.

How can we help students to work mathematically?

To work mathematically, students need to be provided with, and guided through relevant activities. Instead of telling students a formula and giving them a series of rules for its application, teachers

1. begin by providing students with a simple real life context where the concept is used
2. have students investigate similar cases and special cases of the same concept
3. give students hints (when necessary) to help them make linkages between what they need to find out and what they already know
4. help students generalise and formalise what they have learnt (eg as a formula)
5. give students practice in applying what they have learnt to a wide variety problems and practical situations.

What resources are required?

The preparation of lessons that use the Working Mathematically processes to teach mathematical concepts requires considerable time, thought and creativity. Only a few of the Working Mathematically activities listed in the syllabus can be found in textbooks.

To make the Working Mathematically approach of the new syllabus easier for teachers to implement, we have been writing student workbooks with activities that cover every outcome of a content strand. In 2003, we wrote two student workbooks (Part A and Part B) for the Stage 4 Space and Geometry strand (McMaster and Mitchelmore, 2003). These workbooks are being trialled in schools this year. The results of this trial will be reported at the 2005 conference of the Australian Association of Mathematics Teachers Inc.(McMaster and Mitchelmore, 2005). We

have now written two workbooks for the Stage 4 Measurement strand (McMaster and Mitchelmore, 2004) that will be trialled in schools in 2005.

Wherever possible, the Measurement workbooks contain activities that relate to real life or other key learning areas (biology, history, geography, sport, design, agriculture. forestry, the building trades, surveying, astronomy and event scheduling) and all the manipulative materials needed for the activities are inexpensive and readily available.

At the 2004 MANSW conference, teachers were introduced to the Measurement workbooks through a series of activities related to finding the area and side lengths of triangles. One of these activities is given here.

References

Board of Studies (2002). *Mathematics Years 7-10 Syllabus.* Sydney: Author.

McMaster, H., & Mitchelmore, M. (in press). The rewards and difficulties of Working Mathematically. In *Proceedings of the 20th biennial conference of the Australian Association of Mathematics Teachers Inc*. Sydney. January 2005.

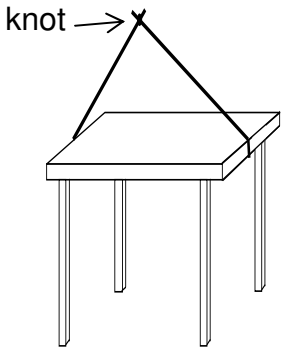
McMaster, H., & Mitchelmore, M. (2004). *Working Mathematically: Measurement Stage 4* (draft edition, two parts). Sydney: Workingmaths.

McMaster, H., & Mitchelmore, M. (2003). *Working Mathematically: Space & Geometry Stage 4* (draft edition, two parts). Sydney: Workingmaths.

The maximum area

Resources required

Each pair of students needs:
a piece of string about 2.5 times the width of a desk
a protractor
a calculator.



Tie a piece of string around the width of your desk as shown, with the knot at the top.

The aim of this activity is to find the maximum area of a triangle that has the desk as its base, the knot as its apex, and string for the remaining two sides.

Person 1

Hold the knot above the desk to make a triangle with string and desktop.
Move the knot from side to side above the desk, keeping the triangle perpendicular to the desk (as seen from your seat).
Try to find the maximum height of the triangle (measured by Person 2).
Copy the measurements recorded by Person 2 into your table below.

Person 2

Measure the height of triangles made by your partner.
When you have found the triangle with the greatest height, record its measurements (to the nearest centimetre) in the table below.

Measurement	Reading (cm)
Perpendicular height (from the knot to the desk)	
Left hand side (string from the desk to the knot)	
Right hand side (string from the desk to the knot)	
Base (width of the desk)	

As you moved the knot, did the perimeter of the triangle change? .....

Why does the triangle with the greatest height have the greatest area?

.....

.....

Calculate the area of this triangle.

.....

.....

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What did you notice about the side lengths of the triangle with the greatest area? .....

What type of triangle is it? .....

A triangle has a perimeter of 18cm. If its base is 8cm, what side lengths will give this triangle the greatest area? .....
Write your answer in the top row of the table below.

Perimeter (cm)	Base (cm)	Side (cm)	Side (cm)
18	8		
18	6		
18	4		

Calculate the side lengths that give the maximum area when the base is 6cm, and then when the base is 4cm. Write your answers in the table.

Draw these triangles on the bases below, using a pair of compasses opened to the required side length to mark the apex.
Label the sides with their lengths.

8 cm

4 cm

6 cm

Measure and label the altitude of each triangle.
Find the triangle with the maximum area.
What length are the sides of this triangle? ....., ... and .....
What type of triangle is it?

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