Working Mathematically in Space and Geometry

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The new K-10 mathematics syllabus has six strands, one if which is entitled “Working Mathematically”. The objective of this strand is:

> Students will develop knowledge, skills and understanding through inquiry, application of problem-solving strategies including the selection and use of appropriate technology, communication, reasoning and reflection. (Board of Studies, 2002, p. 12)

The Working Mathematically strand is clearly derived from two of the strands in *A National Statement of Mathematics for Australian Schools*, namely Mathematical Enquiry, and Choosing and Using Mathematics. Similar strands were included in previous K-10 mathematics syllabuses where they were called Problem Solving in Years 7-8 (Board of Secondary Education NSW, 1990) and Mathematical Investigations in Years 9-10 (Board of Studies, 1996).

The crucial novelty in the new syllabus is that, instead of being taught as a separate strand, Working Mathematically is fully integrated with every other strand. It is, in fact, the core of the new syllabus: Working Mathematically activities are given in relation to every syllabus outcome. Working Mathematically is therefore not something to be done on Fridays or at the end of term, but a method of teaching and learning which can be adopted in every lesson.

What is “Working Mathematically”?

Students are working mathematically when they are learning through experience rather than being told everything. To learn in this way, students need to be provided with, and guided through relevant activities. The teacher's role is to facilitate learning by encouraging students to use the five processes of Working Mathematically that are mentioned in the syllabus, namely *questioning*, *communicating*, *reasoning*, *reflecting* and *applying strategies*.

The Working Mathematically strand has two major aims:

(1) *Thinking*. Working Mathematically activities should teach students how to think for themselves. Students are expected to explore a variety of mathematical situations, make conjectures, and argue for and against these conjectures until they reach a generally accepted conclusion. In the syllabus document, this aspect is embodied mainly in the processes of *communicating* and *reasoning*.

Working Mathematically treats mathematics as a set of interesting challenges or problems rather than as a series of methods and formulae to be learnt for examinations. Students are expected to develop a variety of methods for solving problems and to select the most appropriate one for any given task. The intention of Working Mathematically is that, by thinking for themselves, students will gain a deeper understanding of the mathematics. As the 17th century mathematician Blaise Pascal said:

> We are usually convinced more easily by reasons we have found ourselves than by those which have occurred to others.

(2) *Relevance*. Working Mathematically activities should demonstrate the relevance of mathematics. Many activities are designed to show that mathematics is not abstract and meaningless, but real and useful. Some activities reveal the presence of mathematical concepts in our physical and cultural environment. Others show how mathematical reasoning and results can be applied in everyday life. The processes of *reflecting* and *applying strategies* relate to this second aim.

The fifth process, *questioning*, is central to both aims of Working Mathematically. Questions like "Why does this work?" and "What if we did that?" stimulate thought. In solving challenges, students ask questions of themselves and each other. The use of real life problems, rather than obviously contrived problems, answers students' perennial question "Why are we doing this?" Of course, questioning will only receive such attention when the teacher establishes a classroom culture in which good questions are valued and asking questions is as important, if not more important, than giving correct answers. Jostein Gaarder, in her recent children's storybook *Hello, Is Anybody There?*, compares a good question to a correct answer. She writes:

> An answer is always a stretch of road that lies behind you. Only a question can point the way forward.

The two aims of Working Mathematically are predicated on the assumption that students learn mathematics best by making sense of it—both in terms of internal, logical relations and external, social significance. The intended consequences are that students find mathematics enjoyable and that they develop self-confidence to use mathematics wherever appropriate.

The Space and Geometry strand

Whereas we once had Space in the primary school and Geometry in the secondary school, the new syllabus has a single Space and Geometry strand. This change emphasises the continuity of learning over years K-10, with no sudden shift between Years 6 and 7. It also emphasises that geometry develops out of students' experience of space: geometry is a formal, organised way of looking at the spatial properties which have already been learned through trial and error.

In teaching the Space and Geometry strand, Working Mathematically is especially valuable for (a) drawing students' attention to general spatial properties—the *relevance* aim, and (b) helping students construct logical relationships between them—the *thinking* aim. The following activities specified in the new syllabus for Working Mathematically in Stage 4 of the Space and Geometry strand exemplify these two aspects:

- Interpret and make models from isometric drawings (*Communicating)*
- Apply angle and parallel line results to determine properties of two-dimensional shapes (*Applying Strategies, Reasoning, Reflecting*)
- Bisect an angle by applying geometrical properties eg constructing a rhombus (*Applying Strategies)*
- Justify their solutions to problems by giving reasons using their own words *(Reasoning)*

- Recognise that similar and congruent figures are used in specific designs eg works by Escher and Mondrian (*Reflecting)*

Visual situations provide an attractive context in which to explore mathematical relations, and many students who find algebraic or even arithmetical thinking difficult can excel in spatial explorations. There is a wide variety of properties which can be investigated, and our built and cultural environments provide rich applications. There is therefore a great deal to be gained from Working Mathematically in the Space and Geometry strand.

Teaching Working Mathematically in Space and Geometry

Effective Working Mathematically activities challenge students and guide them to reach important conclusions. Such activities cannot be too structured, or students follow them point-by-point and do not learn to think for themselves; but neither can they be too open-ended, or students will not know what to do and may just waste time.

To provide such activities in relation to the Space and Geometry strand, several resources are needed:

(1) *Hands-on materials.* Spatial explorations inevitably involve constructing shapes in two or three dimensions. For this, one needs a good supply of hands-on materials like straws, pipe cleaners, cardboard, staplers, plasticine and cubes.

(2) *Technology.* The only spatial technology used until recently has been geometrical instruments. These can be slow, inaccurate and dangerous. Dynamic geometry software offers a safer and more efficient way of generating and testing spatial conjectures.

(3) *Ideas.* A large number of Working Mathematically dot points specify particular types of activities. Other ideas may be derived from such sources as *Reflections* and the annual MANSW conferences. Textbooks have some ideas, and more can be gleaned from various websites.

(4) *Worksheets.* Working Mathematically requires students to record the results of investigations in tables, represent information graphically, plot points and lines on grids and make geometric constructions. This is difficult to do if students only have lined exercise books. Worksheets are an absolute necessity.

Of these four resources, some are readily available but others are not. A wide variety of hands-on materials can be purchased quite cheaply from department stores and discount stores, and the dynamic geometry software WinGeom can be downloaded from the internet completely free of charge. The supply of ideas is quite overwhelming, but they are isolated and often seem to lead nowhere. The standard textbooks have some ideas but sections of textbooks labelled as "Working Mathematically" or "Investigation" are sometimes only fact-finding exercises. Worksheets and workbooks that adequately cover all the Working Mathematically activities specified in the new syllabus are not easy to find.

It was such considerations which led the two of us to put together a sequence of worksheets into two workbooks which we have called *Working Mathematically: Space and Geometry, Stage 4*

(McMaster & Mitchelmore, 2003). The draft version is printed in two booklets of about 140 pages each, and covers all the dot points in the SGS4.1 to SGS4.4 outcomes. Part A covers all the basic 2D geometry. Part B revises this content using WinGeom, extends it to complete the Stage 4 2D geometry, and covers all the Stage 4 3D geometry.

The starting point for most activities is a diagram or photograph which students measure, draw on, or cut out and manipulate. In other cases, cheap hands-on material is used—pieces of drinking straws threaded on pipe cleaners being a particular favourite. A large number of illustrations of traditional 2D patterns are included, together with examples of Escher and Mondrian art (reproduced with permission). Pictures of real objects are used to illustrate 3D shapes. The activities are designed to be engaging as well as sequential. It is envisaged that the workbooks could be used with any of the old or new textbook series.

To give a flavour for the sort of activities involved, we give two examples below (one from Part A and one from Part B). They are specifically directed at the following two Working Mathematically dot points:

- Find examples of similar and congruent figures embedded in designs (*Reflecting*)
- Identify parallel, perpendicular and skew lines in the environment (*Communicating, Reflecting*)

Working Mathematically activities can be frustrated when students look up answers at the back of the book. Therefore, no answers are provided in *Working Mathematically: Space and Geometry, Stage 4*. Instead, typical solutions will be put on a password-protected part of the web site www.workingmaths.net and passwords will only be made available to teachers.

Further information

A trial of *Working Mathematically: Space and Geometry Stage 4* is being conducted in Years 7 and 8 at a restricted sample of schools in 2004. A few primary teachers are also trialling selected activities from Part A. Several teachers have contributed to the draft version and we hope that many more will take the opportunity to contribute to a revised edition. Further *Working Mathematically* workbooks for other strands and other stages are also under consideration.

Meanwhile, interested teachers may obtain single copies of the draft booklets from the authors. For further information, please consult the Workingmaths website www.workingmaths.net or send an email to authors@workingmaths.net.

References

Australian Education Council (1990). *A National Statement on Mathematics for Australian Schools.* Melbourne: Curriculum Corporation.

Board of Secondary Education (1990). *Mathematics Syllabus Years 7-8.* Sydney: Author.

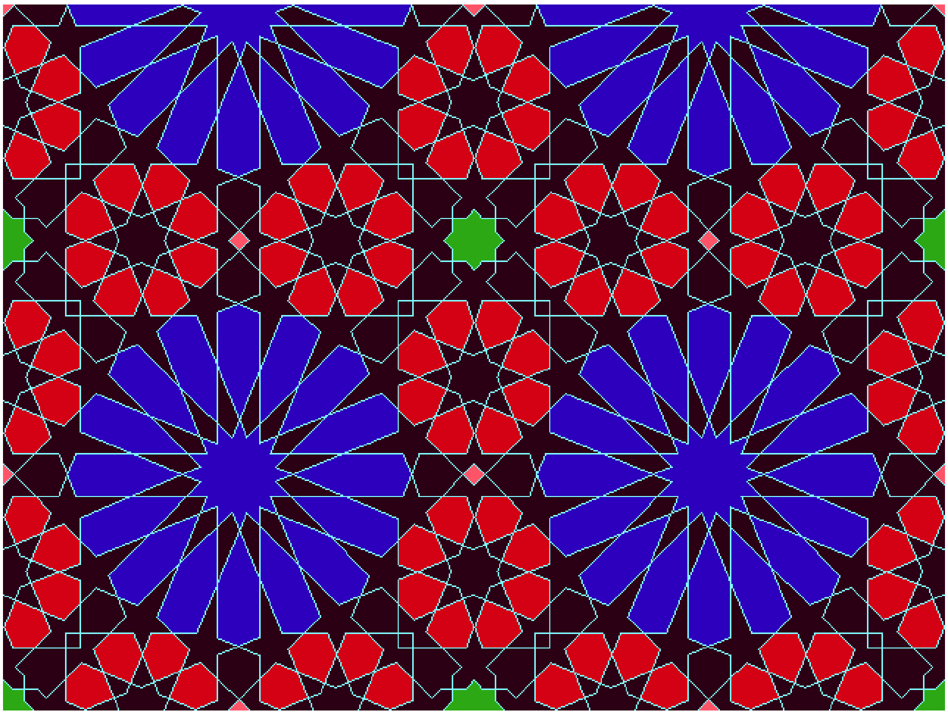
Board of Studies (1996). *Mathematics Years 9-10 Syllabus.* Sydney: Author.

Board of Studies (2002). *Mathematics Years 7-10 Syllabus.* Sydney: Author.

McMaster, H., & Mitchelmore, M. (2003). *Working Mathematically: Space and Geometry Stage 4* (draft edition, two parts). Sydney: Workingmaths.

Activity 4-2: Polygons in Islamic art and design

Below is the historic Spanish Islamic design "Alhambra-Grenada" (14th century). It is based on a square grid.



See this art in colour at http://users.belgacom.net/gc169763/.

Use the square grid below to create your own geometrical design. Colour polygons with the same shape, the same colour. [A square grid is provided.]

Activity 13-6: The surface and lines of a netball court

Form a group of three or four students to answer the following questions.

Imagine that your school has built a new asphalt netball court.

Your group has been given:

- a ball of string
- a sheet of A4 paper
- a pencil.

With these items you are to test the following:

1. whether the court is a plane surface
2. whether the goal post is vertical
3. whether the court is horizontal
4. whether the goal post is in the middle of the back line
5. whether the goal shooting area is a semicircle
6. whether the side lines are parallel to each other.

As a group, think about what things you would use and how you would use them.

Write notes and draw diagrams in the spaces provided below.

Present your ideas to the class.

[Space is provided for students to write their answers.]