15. PRISMS AND CYLINDERS

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Activity 15 – 1

Drawing prisms

AA’∥ BB’ because the points A and B have been translated in the same direction.
AB ∥ A’B’. AA’ and BB’ are parallel (see above) and equal (A and B having been translated the same distance). AB and A’B’ therefore form opposite sides of a parallelogram.

The diagram above looks like the diagram of a triangular prism.

Dashed lines showing hidden edges make the figure above look like a solid.

All the side faces of a prism are parallelograms.
The regular tetrahedron is a Platonic solid that is a prism.

The base of DEFD’E’F’ is the face DEF.
The top of DEFD’E’F’ is the face D’E’F’.
The side faces of DEFD’E’F’ are DEE’D’, EFF’E’ and DFF’D’.
The 3 prisms drawn above are:
Prism A: a pentagonal prism
Prism B: a rectangular prism (possibly a square prism)
Prism C: an octagonal prism

The drawing is an orthographic projection.

No. From the drawing it is not possible to tell whether or not the faces of a prism are perpendicular to its base.
Activity 15 – 2

Modelling prisms

Yes. The edges remain parallel when you change a prism from right to oblique.

Right rectangular prism

All the side faces of a right prism are rectangles.

Both 120° angles and 60° angles in the 2D diagrams above, are right angles in the 3D shapes they represent

Yes. A right triangular prism can have a base that is not a right-angled triangle.
Oblique rectangular prism

Of the four prisms drawn:

- Right prisms are the more stable
- Triangular prisms are the stronger.

Oblique triangular prism

All the side faces of an oblique prism are parallelograms
Activity 15 – 3

Cross-sections of prisms

Section A

Section B

Section C

Section B is parallel to the base. 
Section B is congruent to the base.

Yes. The plane figure (ie. 2D figure) outlined by the thread is always congruent to the base.
Yes. When the thread is parallel to the base, the plane figure outlined by the thread is always congruent to the base, even in an oblique prism. 

All cross-sections of a prism are congruent because a cross-section is a translation of the base.
Activity 15 – 4

Nets of prisms

The nets of parallelepipeds are nets D, E and F. A parallelepiped has 6 faces.

The net of a cuboid is net F.

<table>
<thead>
<tr>
<th>Label of net</th>
<th>Shape of base</th>
<th>Name of prism</th>
<th>Is it right or oblique?</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>parallelogram</td>
<td>parallelogram prism</td>
<td>oblique</td>
</tr>
<tr>
<td>D</td>
<td>square</td>
<td>square prism</td>
<td>oblique</td>
</tr>
<tr>
<td>D</td>
<td>rectangle</td>
<td>rectangular prism</td>
<td>oblique</td>
</tr>
<tr>
<td>E</td>
<td>parallelogram</td>
<td>parallelogram prism</td>
<td>oblique</td>
</tr>
<tr>
<td>E</td>
<td>rhombus</td>
<td>rhombic prism</td>
<td>oblique</td>
</tr>
<tr>
<td>F</td>
<td>rectangle</td>
<td>rectangular prism</td>
<td>right</td>
</tr>
</tbody>
</table>

They are all nets of prisms because they have 2 congruent faces that are a base and a top, and all the other sides are parallelograms.

<table>
<thead>
<tr>
<th>Label of net</th>
<th>Shape of base</th>
<th>Name of prism</th>
<th>Is it right or oblique?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>triangle</td>
<td>triangular prism</td>
<td>oblique</td>
</tr>
<tr>
<td>B</td>
<td>triangle</td>
<td>triangular prism</td>
<td>right</td>
</tr>
<tr>
<td>C</td>
<td>pentagon</td>
<td>pentagonal prism</td>
<td>oblique</td>
</tr>
</tbody>
</table>
Activity 15 – 5

Euler’s formula

<table>
<thead>
<tr>
<th>Type of Prism</th>
<th>Number of sides of base</th>
<th>Number of side faces of prism</th>
<th>Total number of faces of prism</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular</td>
<td>3</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Quadrilateral</td>
<td>4</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Hexagonal</td>
<td>6</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Octagonal</td>
<td>8</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

If the letter 'n' represents the number of sides of the base of a prism:
- The prism has \( n \) side faces.
- The prism has \( n+2 \) faces altogether, so \( F = n+2 \)

<table>
<thead>
<tr>
<th>Type of prism</th>
<th>Number of sides of base</th>
<th>Number of vertices of prism</th>
<th>Number of edges of prism</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular</td>
<td>3</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>Quadrilateral</td>
<td>4</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>Hexagonal</td>
<td>6</td>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>Octagonal</td>
<td>8</td>
<td>16</td>
<td>24</td>
</tr>
</tbody>
</table>

If the base of a prism has \( n \) sides \( V = 2n \)
Its number of edges \( E = 3n \)
\( F = n+2 \)
\( F + V = n+2+2n = 3n+2 \) \( E + 2 = 3n+2 \)
Yes. \( F + V = E + 2 \) so Euler’s formula applies.
Activity 15 – 6

Stacking prisms

Number of prisms in stack = 9

Number of prisms in stack = 12

Number of prisms in stack = 12

Number of prisms in stack = 21
Activity 15 – 7

Cylinders

A cylinder has 2 plane faces and one curved face \( F = 3 \)
A cylinder has no vertices (\( V \)). \( V = 0 \)
A cylinder has 2 edges. \( E = 2 \)
No. Euler's formula does not apply to cylinders. It only applies to convex polyhedra.

No. Cylinders do not tessellate.

A cylinder has an infinite number of planes of symmetry.
A cylinder has one axis of symmetry.

Net \( F \) is the net of a right cylinder.
Net \( E \) is the net of an oblique cylinder.
Activity 15 – 8

Why is it so?

Dice are cubes. They are this shape because this gives them an even probability of landing on any face when rolled.

Books are rectangular prisms. They are this shape because they tessellate when stacked in piles or on shelves.

Toblerones are triangular prisms. They are this shape because it gives them uniqueness as well as a strong box and they tessellate when stacked.

Wheels are cylinders. They are this shape because they roll in a straight line, they are strong and their axis is a constant distance from the perimeter.

CD cases are nearly square prisms. They are this shape because they hold a circular CD and they tessellate when stacked.
Most city buildings are prisms because they are relatively cheap to construct. For a given height, rectangular prisms give the maximum volume of a building on a rectangular block of land.

City towers (eg. Centrepoint Tower in Sydney) are cylindrical because, being so tall, they need to be strong. Also their shape enables a rotating restaurant to be built near the top.

Most buildings are right rather than oblique prisms or cylinders because they are less likely to fall over.

Soup cans are cylinders. They are this shape because they are strong, easy to manufacture, and have a smaller ratio of surface area to volume than do rectangular prisms.

Water pipes are open cylinders. They are this shape because they are strong, easy to manufacture, and have a large cross-sectional area for water to pass through.